Frequency-Domain Approach to Hopf Bifurcation Analysis

Continuous Time-Delayed Systems

This book is devoted to the study of an effective frequency-domain approach, based on systems control theory, to compute and analyze several types of standard bifurcation conditions for general continuous-time nonlinear dynamical systems. A very rich pictorial gallery of local bifurcation diagrams for such nonlinear systems under simultaneous variations of several system parameters is presented. Some higher-order harmonic balance approximation formulas are derived for analyzing the oscillatory dynamics in small neighborhoods of certain types of Hopf and degenerate Hopf bifurcations.

The frequency-domain approach is then extended to the large class of delay-differential equations, where the time delays can be either discrete or distributed. For the case of discrete delays, two alternatives are presented, depending on the structure of the underlying dynamical system, where the more general setting is then extended to the case of distributed time-delayed systems. Some representative examples in engineering and biology are discussed.

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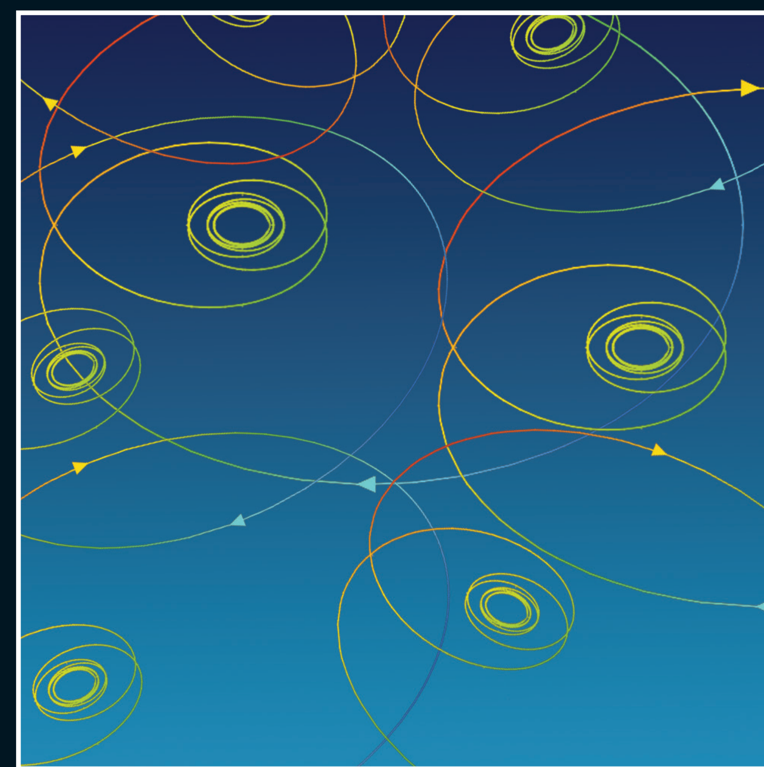
Franco Sebastián Gentile, Jorge Luis Moiola & Guanrong Chen



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Preface

The fundamental theory of periodic solutions (in particular, limit cycles) of nonlinear ordinary differential equations (ODEs) was mainly attributed to the great French mathematician Jules Henri Poincaré (1854-1912). The idea of representing the dynamics of a nonlinear ODE in the phase-space by using what is called the Poincaré return map today, and the preliminary results of the limit cycles theory such as their existence and characteristics, are just a few pieces of the most valuable legacies that Poincaré left to the modern scientific and engineering communities. The significance and generality of his profound analysis has made an extraordinary impact on the theories of nonlinear ODEs and dynamical systems, and has greatly motivated his successors in the pursuit of the modern nonlinear sciences.

For two-dimensional ODE systems, the earlier conjecture about the existence of periodic solutions given by Poincaré was formally presented by the Soviet mathematician A. A. Andronov and his colleagues. Ever since then, this existent result for periodic solutions of two-dimensional ODEs has been referred to as the Poincaré-Andronov conjecture in the literature. Independently, the German mathematician E. Hopf published in 1942 an elegant result that shows the existence of limit cycles in n-dimensional ODE systems, for $n \geq 2$, assuming only the smoothness of the nonlinear vector fields of the systems. This is the celebrated Hopf bifurcation theorem. Basically, the theorem proves that the amplitude and frequency of a periodic solution of such a system can be approximately calculated when a key real parameter of the system is varied. In addition, the theorem explains how the stability of the periodic solution, which is bifurcating from the equilibrium, can be determined as the key parameter is varied. Then, the Hopf bifurcation theorem was reconfirmed and applied about thirty years later by many other researchers from different disciplinary fields when some powerful computational tools became available.

All the aforementioned works use the state-space formulation, namely, a system of nth-order ordinary differential equations. This will be referred to as the "time domain" approach in this book. Yet, there is another interesting formulation of the same dynamical systems available in the literature. This alternative representation applies the familiar engineering feedback systems theory and methodology: an approach described in the "frequency domain", the complex domain after the standard Laplace transforms have been taken on the time-domain state-space systems. The frequency-domain approach was initiated by Allwright, Mees and Chua in the late 1970s. This methodology has an enjoyable engineering flavor and, indeed, possesses several advantages over the classical time-domain methods. A typical one is its pictorial characteristic that utilizes advanced computer graphical capabilities and so bypasses quite a lot of profound and difficult mathematical analysis, specially for some infinite-dimensional systems. As a result, it clearly visualizes some fairly complex dynamical behavior. This book is devoted to this frequency-domain approach, for both regular and degenerate Hopf bifurcation theories for continuous-time systems, including those with time delays.

Proceeding with thorough discussions in the following chapters, the reader will be able to realize that many significant results and computational formulas obtained in the studies of regular and degenerate Hopf bifurcations from the time-domain approach can also be reformulated into the corresponding frequency-domain setting, and be rediscovered by using the frequency-domain methods.

This book shows in detail how the frequency-domain approach can be used to obtain several types of standard bifurcation conditions for general nonlinear dynamical systems. A rich pictorial gallery of local bifurcation diagrams for nonlinear systems under simultaneous variations of several system parameters is demonstrated. In addition, in conjunction with this graphical analysis of local bifurcation diagrams, the defining and non-degeneracy conditions for several degenerate Hopf bifurcations are presented. Some higher-order harmonic balance approximation formulas are derived for analyzing the dynamical behavior in small neighborhoods of certain types of degenerate Hopf bifurcations. These formulas allow one to better approximate the amplitudes and frequencies of oscillations that are "far away" from an equilibrium. With these improved approximations, those limit cycles containing an important harmonic content (i.e., when the amplitudes of harmonics have comparable sizes with respect to the amplitude of the main frequency component) can be described more accurately.

The frequency-domain approach is also extended to the case of delay-differential equations, where the time delays can be discrete or distributed. For the case of discrete delays, several alternative analysis methods are presented, depending on the structure of the system under study. A general alternative is then extended to the case of distributed delay systems, for several models mainly concerning biological applications.

The book is organized as follows. In Chapter 1, some fundamental mathematical concepts and results about stability and bifurcations in nonlinear dynamical systems are reviewed. Some classical analysis tools are briefly introduced. In Chapter 2, the frequency-domain approach to the Hopf bifurcation for ODEs is introduced. The advantages as well as limitations of this frequency-domain approach are commented and some illustrative examples are presented. Chapter 3 is devoted to the analysis of static and multiple bifurcations points using the graphical methods and tools. The most elementary static bifurcations are presented and their computation under the frequency-domain framework is demonstrated. Chapter 4 is devoted to the study of some explicit formulas that can be used as efficient conditions for recovering the degenerate (i.e., singular) bifurcation points of a dynamical system under simultaneous variations of several system parameters. A graphical method for computing certain singularities that are crucial to the understanding of the global dynamics is developed. Chapter 5 studies an extension of the valid domain for periodic solutions, where higher-order harmonic balance approximations are applied. The results on the continuation of periodic solutions obtained from the frequency-domain approach are verified and compared to those obtained by using a specific software or some other well-known methods. A computational algorithm is derived in this chapter, for the continuation of periodic solutions near degenerate Hopf bifurcation points of certain types. In Chapter 6, the frequency-domain method developed in the previous chapters is applied to detecting oscillations in nonlinear systems with discrete-time delays. As is well known, there will be an infinite number of eigenvalues in the corresponding linearized system, so it can be expected that a great diversity of multiple and degenerate Hopf bifurcations exists, as compared to the nonlinear systems without time delays. Feedback systems that have only one time delay in the linear and/or nonlinear feedback loop are considered. In Chapter 7, a general approach for the study of distributed-delay systems is introduced. This formulation is particularly useful for biological applications. Some interesting examples are discussed. In Chapter 8, different cases of degenerate Hopf bifurcations in time-delay systems are analyzed, some of which have two or even more delays. Finally, some higher-order formulas for the Hopf bifurcation are provided in the Appendices.

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Contents

Preface vii

1. Stability and bifurcation analysis 1

1.1 Preliminaries . . . 1
1.1.1 Equilibrium points and periodic solutions . . . 2
1.1.2 Stability . . . 4
1.1.3 Bifurcations . . . 7
1.2 Overview of bifurcations . . . 13
1.2.1 A single eigenvalue at 0 . . . 13
1.2.2 A pair of complex conjugate eigenvalues . . . 16
1.2.3 The Hopf bifurcation theorem . . . 18
1.2.4 Degeneracies . . . 21
1.3 Classical methods for bifurcation analysis . . . 24
1.3.1 The center manifold theorem . . . 24
1.3.2 The normal form theory . . . 25

2. The Hopf bifurcation theorem in the frequency domain 27

2.1 Introduction . . . 27
2.2 Formulation of the frequency-domain approach . . . 29
2.2.1 Feedback control representation of the system . . . 29
2.2.2 Stability analysis: The Nyquist criterion . . . 31
2.2.3 Choosing an adequate representation . . . 33
2.2.4 Emergence of a periodic solution . . . 34
2.3 Advantages of the frequency-domain approach . . . 41
2.4 Application examples of the graphical Hopf theorem . . . 43
2.4.1 The normal form of the Hopf bifurcation . . . 43

2.4.2 The tunnel-diode oscillator 46
2.4.3 The continuous-flow stirred tank reactor 49

3. Analysis of static and multiple bifurcations 57

3.1 Introduction 57
3.2 Formulation of elementary bifurcation conditions on the parameter plane 58
3.3 Applications of the frequency-domain formulas 62
3.3.1 The saddle-node bifurcation 62
3.3.2 The transcritical bifurcation 64
3.3.3 The hysteresis bifurcation 65
3.3.4 The pitchfork bifurcation 67
3.3.5 Static bifurcations in chemical reactor models . . 69
3.4 Formulation of multiple crossings and determination of degeneracies 72
3.5 Applications of the frequency-domain formulas to multiple bifurcations 76
3.6 Multiplicity of equilibrium solutions in the parameter space 86

4. Degenerate Hopf bifurcations 89

4.1 Introduction 89
4.2 Degenerate Hopf bifurcations of co-dimension one on the parameter plane 91
4.2.1 Local bifurcation diagrams 92
4.2.2 Application examples 98
4.3 Multiple Hopf bifurcation points in the parameter space . 108
4.4 Degenerate Hopf bifurcations and the singularity theory . 121
4.5 Degenerate Hopf bifurcations and feedback systems 132
4.6 Degenerate Hopf bifurcations: The graphical Hopf theorem 139
4.6.1 Degenerate Hopf bifurcations of the H_{0m} type . . 139
4.6.2 Degenerate Hopf bifurcations of the H_{n0} type . . 144
4.7 Some applications 149

5. Higher-order Hopf bifurcation formulas 155

5.1 Introduction 155
5.2 Approximations of periodic solutions by higher-order formulas 156
5.2.1 The algorithm 159

5.2.2 Some applications 160
5.3 Continuation of periodic solutions: degenerate cases . . . 167
5.4 Local bifurcation diagrams: The graphical Hopf theorem . 176
5.5 Algorithms for recovering periodic solutions 179
5.5.1 The original formulation 179
5.5.2 The modified scheme 180
5.5.3 An iterative graphical Hopf method 181
5.5.4 Study of the van der Pol system 184
5.5.5 Harmonic distortion 191

6. Hopf bifurcation in continuous-time systems with discrete-time delays 195

6.1 Introduction 195
6.2 Preliminaries: Retarded functional differential equations . 197
6.2.1 Existence and uniqueness of solutions 199
6.2.2 Stability notions for retarded functional differential equations 199
6.2.3 Linear delay-differential equations 200
6.2.4 Linearised stability criterion 202
6.3 Hopf bifurcation in retarded functional differential equations 206
6.4 The graphical Hopf bifurcation theorem and delay-differential equations 209
6.4.1 Feedback systems with a single delay in the loop . 209
6.4.2 Systems with delay in the linear block and in the nonlinear feedback 214
6.4.3 An alternative approach 226
6.5 Some applications 237
6.5.1 Dynamics of baroreflex control of heart rate . . . 237
6.5.2 Internet congestion control 245
6.6 Analysis of delay equations of the neutral type 260

7. Hopf bifurcation in continuous-time systems with distributed time delays 269

7.1 Introduction 269
7.2 Discrete versus distributed delays 271
7.3 A common approach: equivalent models 274

7.4 The graphical Hopf method for distributed delay systems: The general case . . . 276
7.5 Application examples . . . 282
7.5.1 A simple scalar system . . . 282
7.5.2 A system of coupled neurons . . . 295

8. Degenerate bifurcations in time-delayed systems 309
8.1 Conditions for degenerate bifurcations in time-delayed systems . . . 309
8.2 Applications . . . 313
8.2.1 A nonlinear feedback control system with two time-delays . . . 313
8.2.2 Variable structure control and the Smith predictor 315
8.2.3 Cascading time-delayed feedback integrators . . . 316
8.2.4 Analysis of the 1 : 2 resonant double Hopf bifurcation . . . 324
8.2.5 Campbell–LeBlanc system . . . 324

Appendix A Higher-order Hopf bifurcation formulas: Part I 337

Appendix B Higher-order Hopf bifurcation formulas: Part II 347

Appendix C Higher-order Hopf bifurcation formulas: Part III 349

Bibliography 351

Subject Index 369

Author Index 373